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Maximum Likelihood Method Based on Interior Point Algorithm for Aircraft Parameter Identification

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I. Introduction

SYSTEM identification has extensive applications in aircraft's aerodynamics analysis and design. For aircraft's aerodynamic parameter identification, maximum likelihood methods are most frequently used.[1–3] Provided that the model structure is known and there are sufficient sampled data, the true value of the parameters can be precisely identified by maximum likelihood methods. Some traditional maximum likelihood methods are well established in the literature, such as the output error method,[1,2] which considers only the measurement noise, and the filter error method,[3,4] with the process and the measurement noise considered. These methods run the risk of being trapped in a local minimum outside the reasonable range due to the local optimization ability. When an active set strategy is used and there is knowledge of the constraints on the unknown parameters, a so-called bounded variable maximum likelihood method is proposed to keep the identified results in a reasonable range.[2] However, the optimizer of the bounded variable maximum likelihood method[2] is still a Gauss–Newton type, as is the optimizer adopted in Ref. 1. It is well known that an interior point algorithm is a computationally powerful tool for mathematical programming problems.[5] A solution based on the interior point algorithm is given to the state estimation problem with algebraic constraints on unknown parameters.[6,7] The method is limited in that the object considered in Refs. 6 and 7 is an algebraic system, whereas in aircraft parameter identification the object is generally a dynamic system. In this paper, an alternative method is proposed to identify unknown parameters of a nonlinear system with some information available, which uses the interior point algorithm as an optimizer and ensures that the identification algorithm always gives a reasonable result.

II. Maximum Likelihood Method Based on Interior Point Algorithm

A generic dynamic system is represented as follows:

$$\dot{\boldsymbol{x}}(t) = f[\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{\theta}], \qquad \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \tag{1}$$

$$\boldsymbol{y}(t) = g[\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{\theta}] \tag{2}$$

$$z(t_k) = \boldsymbol{y}(t_k) + \boldsymbol{v}(t_k), \qquad k = 1, 2, \ldots, N \tag{3}$$

where $\boldsymbol{x}(t)$ is the system state vector, $\boldsymbol{u}(t)$ is the system input vector, and $\boldsymbol{y}(t)$ is system output vector. Here $\boldsymbol{z}$ is the measurement data disturbed by a Gaussian random noise vector $\boldsymbol{v}$ with an unknown noise covariance matrix R and $\boldsymbol{\theta}$ is an unknown parameter.

We are concerned with the problem of identifying the unknown parameter $\boldsymbol{\theta}$ and noise covariance matrix R. Using the measurement data, we construct the maximum likelihood cost function as follows:

$$J(\theta) = \frac{1}{2}\sum_{k=1}^{N}[\boldsymbol{z}(t_k) - \boldsymbol{y}(t_k, \boldsymbol{\theta})]^T R^{-1}[\boldsymbol{z}(t_k) - \boldsymbol{y}(t_k, \boldsymbol{\theta})] + \frac{N}{2}\ln|R| \tag{4}$$

In practice, some transcendental knowledge is always available, such as the range of unknown parameters. We can formulate the parameter identification problem in Eq. (4) as the following optimization problem:

$$\min_{\theta} J(\boldsymbol{\theta}) \text{ subject to } c(\boldsymbol{\theta}) \le 0 \tag{5}$$

where $c(\boldsymbol{\theta})$ is a first-order differentiable function of $\boldsymbol{\theta}$ with known form, which represents the transcendental knowledge of the system about $\boldsymbol{\theta}$.

The optimization problem in Eq. (5) can be solved by a logarithmic barrier function method. First, it can be transformed into the next optimization problem:

$$\min_{\theta} J(\boldsymbol{\theta}) \text{ subject to } c(\boldsymbol{\theta}) + \boldsymbol{s} = 0, \qquad \boldsymbol{s} \ge 0 \tag{6}$$

where $\boldsymbol{s}$ is a slack variable. To build our algorithm, we consider the following version of Eq. (6):

$$\min_{\theta} \hat{J}(\boldsymbol{\theta}) = \min_{\theta} J(\boldsymbol{\theta}) + \boldsymbol{\lambda}^T[c(\boldsymbol{\theta}) + \boldsymbol{s}] - \mu\sum_{i=1}^{p}\ln s_i \tag{7}$$

where $\boldsymbol{\lambda}$ is the Lagrange multiplier, μ is the barrier parameter, s_i is the ith element of $\boldsymbol{s}$, and p is the dimension of inequality constraints. If $\mu > 0$, Eq. (7) is equivalent to

$$\min_{\theta} \hat{J}(\boldsymbol{\theta}) = \min_{\theta} J(\boldsymbol{\theta}) + \boldsymbol{\lambda}^T\sqrt{\mu}[c(\boldsymbol{\theta}) + \boldsymbol{s}] - \mu\sum_{i=1}^{p}\ln s_i \tag{8}$$

From the first-order optimality conditions for Eq. (8), we have the following:

$$R = \frac{1}{N}\sum_{k=1}^{N}[\boldsymbol{z}(t_k) - \boldsymbol{y}(t_k)][\boldsymbol{z}(t_k) - \boldsymbol{y}(t_k)]^T \tag{9}$$

$$-\sum_{k=1}^{N}\left[\frac{\partial y(t_k)}{\partial\theta}\right]^T R^{-1}[z(t_k) - \boldsymbol{y}(t_k, \theta)] + C^T\sqrt{\mu}\lambda = 0 \tag{10}$$

$$c(\boldsymbol{\theta}) + \boldsymbol{s} = 0 \tag{11}$$

$$S\boldsymbol{\lambda} - \sqrt{\mu}\boldsymbol{e} = 0 \tag{12}$$

where $C = \partial c(\boldsymbol{\theta})/\partial\boldsymbol{\theta}$, $S = \text{diag}\{s_1, \ldots, s_p\}$, and $\boldsymbol{e} = [1, \ldots, 1]^T \in \Re^p$.

Theorem: Suppose that μ converges to zero, $\boldsymbol{s} \ge 0$, and $\boldsymbol{\lambda} \ge 0$. Equations (10–12) are the necessary conditions for minimizing the maximum likelihood function in Eq. (4).

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Proof. Equations 10–12 can be expressed as

$$\frac{\partial[J(\boldsymbol{\theta})+\boldsymbol{\lambda}^T\sqrt{\mu}c(\boldsymbol{\theta})]}{\partial\boldsymbol{\theta}}=0 \tag{13}$$

$$\lambda_i c_i(\boldsymbol{\theta})=-\sqrt{\mu}, \qquad i=1,\ldots,p \tag{14}$$

$$\boldsymbol{s}\geq 0\,[c(\boldsymbol{\theta})\leq 0], \qquad \boldsymbol{\lambda}\geq 0 \tag{15}$$

If μ approaches zero, Eq. (14) can be expressed as

$$\lambda_i c_i(\boldsymbol{\theta})=0, \qquad i=1,\ldots,p \tag{16}$$

Equations (13), (15), and (16) are the Karush–Kuhn–Tucker (KKT) conditions (see Ref. 8) for optimality of Eq. 4. This concludes the proof.

Perturbing the Eqs. 10, 11 and 12, we have

$$\Delta\theta=M^{-1}\left(\sum_{k=1}^{N}\left[\frac{\partial y(t_k)}{\partial\theta}\right]^T R^{-1}[z(t_k)-y(t_k)] - C^T S^{-1}\hat{\lambda}\left\{\sqrt{\mu}[\boldsymbol{s}+c(\boldsymbol{\theta})]+\mu\hat{\lambda}^{-1}\boldsymbol{e}\right\}\right) \tag{17}$$

$$\Delta\boldsymbol{s}=-C\Delta\boldsymbol{\theta}-c(\boldsymbol{\theta})-\boldsymbol{s} \tag{18}$$

$$\Delta\boldsymbol{\lambda}=S^{-1}\left(\sqrt{\mu}\boldsymbol{e}-S\boldsymbol{\lambda}-\hat{\lambda}\Delta\boldsymbol{s}\right) \tag{19}$$

where

$$\hat{\lambda}=\mathrm{diag}\{\lambda_1,\ldots,\lambda_p\}$$

$$M=\sum_{k=1}^{N}\left[\frac{\partial y(t_k)}{\partial\theta}\right]^T R^{-1}\left[\frac{\partial y(t_k)}{\partial\theta}\right]+\sqrt{\mu}C^T S^{-1}\hat{\lambda}C$$

and μ is selected to guarantee that it converges to zero as follows.

1) If $(\boldsymbol{\lambda}^{j+1})^T\boldsymbol{s}^{j+1}\leq\mu^j p(n+p)$ is held, then

$$\mu^{j+1}=\frac{(\boldsymbol{\lambda}^{j+1})^T\boldsymbol{s}^{j+1}}{(n+p)^2} \tag{20}$$

where μ^{j+1}, $\boldsymbol{\lambda}^{j+1}$, and $\boldsymbol{s}^{j+1}$ are estimation values of variables μ, $\boldsymbol{\lambda}$, and $\boldsymbol{s}$ at the $(j+1)$th iteration and n is the dimension of $\boldsymbol{x}$.

2) For a given integer $\bar{N}$, if $(\boldsymbol{\lambda}^{j+1})^T\boldsymbol{s}^{j+1}>\mu^j p(n+p)$ is continuously held only within $\bar{N}$ iteration steps, then

$$\mu^{j+1}=\mu^j \tag{21}$$

3) For a given integer $\bar{N}$, if $(\boldsymbol{\lambda}^{j+1})^T\boldsymbol{s}^{j+1}>\mu^j p(n+p)$ is continuously held more than $\bar{N}$ iteration steps, then

$$\mu^{j+1}=[p/(n+p)]\mu^j \tag{22}$$

Thus, the following fact is obvious.

Fact 1: When $j\to\infty$, $\mu\to 0$.

The following treatment is adopted to ensure the interior property in the process of updating $[\boldsymbol{\theta}+\Delta\boldsymbol{\theta},\boldsymbol{s}+\Delta\boldsymbol{s},\boldsymbol{\lambda}+\Delta\boldsymbol{\lambda}]$:

$$\boldsymbol{\theta}^{j+1}=\boldsymbol{\theta}^j+\alpha_p\Delta\boldsymbol{\theta}^j \tag{23}$$

$$\boldsymbol{s}^{j+1}=\boldsymbol{s}^j+\alpha_p\Delta\boldsymbol{s}^j \tag{24}$$

$$\boldsymbol{\lambda}^{j+1}=\boldsymbol{\lambda}^j+\alpha_d\Delta\boldsymbol{\lambda}^j \tag{25}$$

where α_p and α_d are proper coefficients, selected according to Refs. 5 and 6, to ensure that the following fact holds.

Fact 2: In the iteration of Eqs. (23–25), $\boldsymbol{s}\geq 0$ and $\boldsymbol{\lambda}\geq 0$.

From the theorem, Fact 1, and Fact 2, the identification method proposed in this paper provides a promising way to obtain the true value of unknown parameters. When $\mu>0$, our method will make the identified parameters converge to the true value by the variable μ decreasing to zero and will ensure that the identified results always stay in a reasonable range. Furthermore, if μ approaches zero, the converged results behave like the generic maximum likelihood identification method without considering the constraints on the unknown parameters.

Remark: If μ converges to zero, Eq. (17) reduces to the following equation:

$$\Delta\theta=\left\{\sum_{k=1}^{N}\left[\frac{\partial y(t_k)}{\partial\theta}\right]^T R^{-1}\left[\frac{\partial y(t_k)}{\partial\theta}\right]\right\}^{-1} \times\left\{\sum_{k=1}^{N}\left[\frac{\partial y(t_k)}{\partial\theta}\right]^T R^{-1}[z(t_k)-y(t_k)]\right\} \tag{26}$$

Equation (26) is the general iteration form of the maximum likelihood method, from which it is known that the Fisher information matrix and the identification precision of our method are the same as the generic maximum likelihood method.

III. Application to Aircraft Parameter Identification

To verify the maximum likelihood method based on interior point algorithm with variable constraints, an example of identifying longitudinal derivatives is considered based on simulated flight data. The longitudinal dynamics of an aircraft with elevator are as follows.

The state equations are given by

$$\dot{\alpha}=q+g\cos(\theta-\alpha)/V_0-L/(mV_0) \tag{27}$$

$$\dot{\theta}=q \tag{28}$$

$$\dot{q}=M/I_y \tag{29}$$

where

$$L=\bar{q}S(C_{L0}+C_{L\alpha}\alpha)$$

$$M=\bar{q}S\bar{c}[C_{m0}+C_{m\alpha}\alpha+(\bar{c}/2V_0)C_{mq}q+C_{m\delta e}\delta_e]$$

where α is angle of attack; θ is pitch angle; q is pitch rate; g is acceleration of gravity; $\bar{q}$ is dynamic pressure, S is reference area; $\bar{c}$ is reference chord; I_y is pitching moment of the inertia; m is mass of the aircraft; V_0 is a constant speed; δ_e is elevator angle; C_{L0}, $C_{L\alpha}$, C_{m0}, $C_{m\alpha}$, C_{mq}, and $C_{m\delta e}$ are the unknown longitudinal derivatives of the aircraft.

The output equations are given by

$$\boldsymbol{y}=[\alpha;\theta;q] \tag{30}$$

with measurement equations

$$\boldsymbol{z}(t_k)=\boldsymbol{y}(t_k)+\boldsymbol{v}(t_k) \tag{31}$$

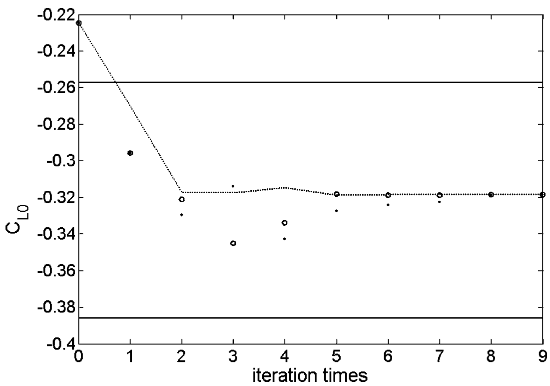
where $\boldsymbol{v}$ is white noise. To excite the system persistently, system input δ_e is chosen as a 3–2–1–1 sequence. In this example, linear inequality constraints on unknown parameters are considered, as well as what are considered in the bounded variable maximum likelihood method in Ref. 2. That is, the unknown parameters θ have upper bounds and (or) lower bounds

$$c(\boldsymbol{\theta})=\left[(\boldsymbol{\theta}-\boldsymbol{\theta}_{\max})^T,(\boldsymbol{\theta}_{\min}-\boldsymbol{\theta})^T\right]^T\leq 0 \tag{32}$$

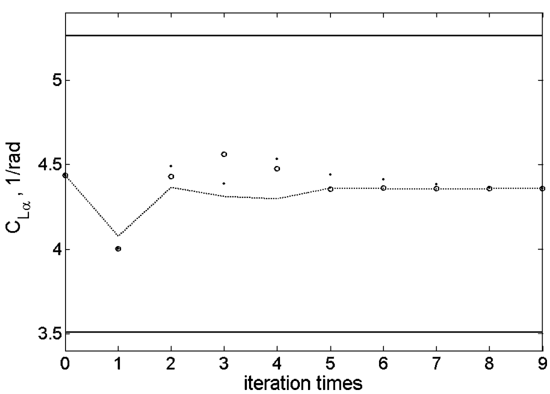
Parameter identification is carried out by applying the conventional unconstrained maximum likelihood method in Ref. 1, the bounded variable maximum likelihood method in Ref. 2, and the maximum likelihood method based on interior point algorithm presented in this Note. The results can be seen in Table 1. The Cramer–Rao bounds for the standard errors are included in Table 1, just behind the identified parameter values. Figure 1 shows the iteration process of parameter identification. In the case of Fig. 1, observe that the maximum likelihood method based on the interior point algorithm provides a smoother variation of the identification results

Table 1 Parameter identification results

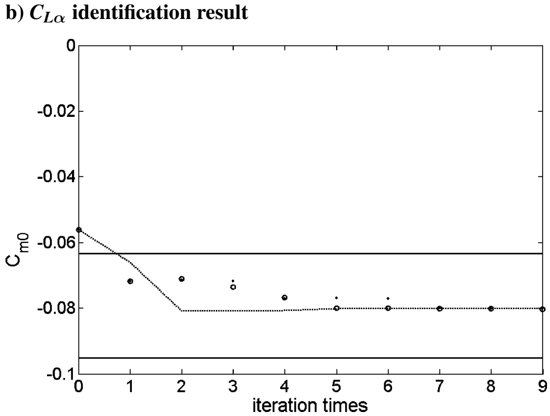
Identified parameters	True value	Method in Ref. 1	Method in Ref. 2	Present method
C_{L0}	−0.3216	-0.3186 ± 0.0157	-0.3184 ± 0.0155	-0.3185 ± 0.0154
$C_{L\alpha}$	4.3863	4.3599 ± 0.1025	4.3596 ± 0.1026	4.3614 ± 0.1024
C_{m0}	−0.0794	-0.0802 ± 0.0022	-0.0803 ± 0.0017	-0.0802 ± 0.0016
$C_{m\alpha}$	−0.2284	-0.2279 ± 0.0024	-0.2280 ± 0.0029	-0.2280 ± 0.0024
C_{mq}	−2.0591	-2.1003 ± 0.2054	-2.1053 ± 0.2020	-2.0886 ± 0.2330
$C_{m\delta e}$	−1.1380	-1.1450 ± 0.0232	-1.1459 ± 0.0188	-1.1448 ± 0.0182



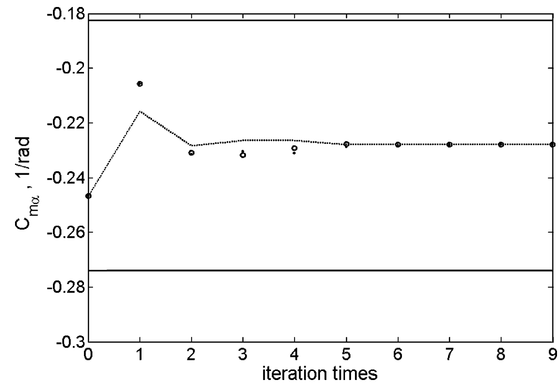
a) C_{L0} identification result



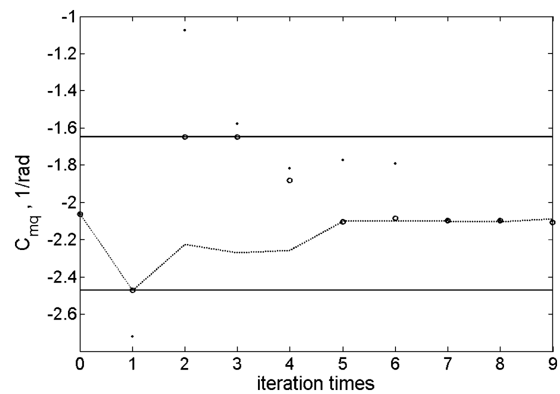
b) $C_{L\alpha}$ identification result



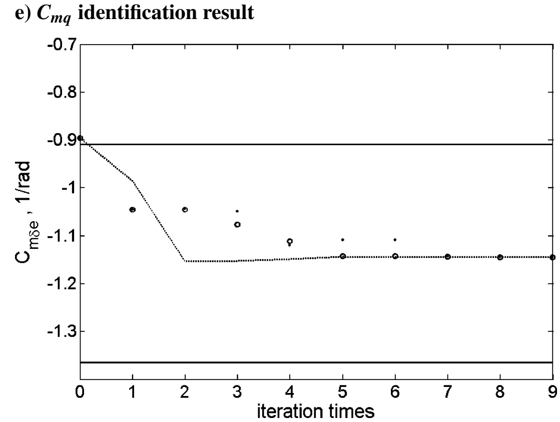
c) C_{m0} identification result



d) $C_{m\alpha}$ identification result



e) C_{mq} identification result



f) $C_{m\delta e}$ identification result

Fig. 1 Parameter identification iteration process: ●, maximum identification method without constraints (Ref. 1); ○, maximum identification method with constraints based on active set strategy (Ref. 2); - - -, maximum identification method with constraints based on interior point algorithm; and ——, bounds of unknown parameters.

and requires fewer iterations for convergence. In the foregoing example, and in some other test cases not presented here, it is shown that our method has comparable identification results and iteration times with the methods in Refs. 1 and 2.

IV. Conclusions

An optimization method based on the interior point algorithm provides a solution to the maximum likelihood identification problem under the constraints of unknown parameters, which fully makes use of the obtained information about the unknown parameters to ensure that the identification results always stay in a reasonable range. Most importantly, the method described in this Note applies the powerful optimization tool, interior point algorithm, to the maximum likelihood identification method. It is proven that our identification method provides a promising way to identify the true value of unknown parameters, using the KKT condition for nonlinear optimization and that the identification precision of the maximum likelihood method based on interior point algorithm is theoretically the same as the generic maximum likelihood method. The proposed method is an efficient alternative to some traditional maximum likelihood methods. A simulation example of identifying the longitudinal derivatives of an aircraft demonstrates that the convergence of the maximum likelihood method based on interior point algorithm presented in this Note may require fewer iterations than that of the generic maximum likelihood method, and it is expected to be theoretically proven in future works.

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